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ELECTRONIC TRACKING ERRORS VERSUS PLASMA EFFECTS

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR- 64-74

AUGUST 1964

J. Hoffman

Prepared for

DIRECTORATE OF AEROSPACE INSTRUMENTATION
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 705. 1

Prepared by

THE MITRE CORPORATION
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FOREWORD

The author would like to express his thanks to Dr. Harry Schecter for reviewing the manuscript.

ELECTRONIC TRACKING ERRORS VERSUS PLASMA EFFECTS

ABSTRACT

Simple models are utilized to calculate the order of magnitude of errors generated by reentry and flame plasma effects in typical tracking systems.

REVIEW AND APPROVAL

Publication of this technical documentary report does not constitute Air Force approval of its findings or conclusions. It is published only for the exchange and stimulation of ideas.

CONTENTS

ELECTRONIC TRACKING ERRORS VERSUS PLASMA EFFECTS

SECTION I

INTRODUCTION

GENERAL

Plasma is generated during important portions of a missile trajectory, namely boost and reentry. The two primary types of plasma generated are flame plasma and reentry plasma. Either type interferes with the electromagnetic linkage of the vehicle with the ground stations. The interference is frequently severe enough to completely break this linkage, causing "blackout." Considerable effort has been, and is continuing to be, focused upon attempts to overcome this blackout.

During a substantial portion of the trajectory, the electromagnetic linkage is maintained, even in the presence of plasma. This is because the plasma properties change throughout the trajectory and, hence, the level of interference also changes. Also, the electromagnetic path length through the plasma changes throughout the trajectory, resulting in variations in the degree of interference.

In this report, we shall be concerned with those portions of the trajectory where the degree of interference is less than blackout. More specifically, we shall investigate the interference in electronic tracking which results in tracking errors.

OBJECTIVES AND SCOPE

During those portions of the trajectory where tracking is successful, the presence of plasma results in numerous tracking errors. We shall investigate those errors which are related to phase and transit time variations produced when

the tracking signal traverses the plasma before reaching the vehicle and before returning to the ground station.

Since the main purpose of this document is to point out the existence of plasma-induced tracking errors and to estimate their magnitudes, extremely simple models will be used. To make a more precise error calculation would require further refinement of the plasma models and also would require consideration of specific vehicles and trajectories; these are outside the scope of this report.

The phase and transit time plasma variations produce errors in:

(a) Continuous wave (CW) range measurements,

(b) Doppler range-rate measurements,

(c) CW angle measurements, and

(d) Pulse measurements.

This report is limited to these types of plasma-generated electronic tracking errors.

SECTION II

PLASMA-INDUCED ERRORS

GENERAL

The basic assumptions and equations describing the plasma-electromagnetic interaction are contained in Reference 1. We shall draw heavily from the results contained in this Reference. Only significant extensions of these results and the derivation of new relationships will be contained in this report.

The presence of a plasma between the vehicle and the tracking station may generate a multitude of different types of errors. We shall concern ourselves with time and phase variations produced by traversing the plasma. These produce errors in the electronic tracking parameters: range, range-rate and angle.

We shall first determine the functional dependence of these errors upon the plasma characteristics, and then apply the results to reentry and flame plasma.

CW RANGE MEASUREMENTS

Let us consider the range error produced when the tracking signal traverses the plasma, reaches the vehicle, is retransmitted, and traverses the plasma again before reaching the ground station. CW measurements of range (GLOTRAC, MISTRAM, etc.,) are based on the fact that the wave experiences a phase shift proportional to the distance travelled. The plasma-induced phase shift differs from the free space phase shift and produces a range error, ΔR.

The phase φ of the wave received by the ground station is

$$\varphi = \omega \left[t - \frac{2}{c} (R + SP) \right], \tag{1a}$$

where

ω = tracking signal angular frequency,

t = time,

R = range,

c = velocity of light in free space,

S = plasma distance traversed, and

P = plasma phase shift factor. (The factor of 2 is due to the round trip.)

The total phase shift $\Delta\varphi$ experienced by the wave is

$$\Delta\varphi = 2 \frac{\omega}{c} (R + SP) . \tag{1b}$$

The range is, therefore, given by

$$R = \frac{1}{2} \frac{c}{\omega} \Delta\varphi - SP . \tag{2}$$

Since the range is, in practice, obtained from Eq. (2) with P = 0, the range error is

$$\Delta R = - SP . \tag{3}$$

The phase shift factor P is, in general, a function of signal frequency and plasma characteristics.

DOPPLER RANGE-RATE MEASUREMENTS

Doppler systems transform received frequency shifts into range-rate data. From Reference 2, the first-order round-trip Doppler equation is

$$\omega_R \approx \omega_T \left(1 - 2 \frac{\dot{R}}{c}\right) , \tag{4a}$$

where

ω_R = received angular frequency,

ω_T = transmitted angular frequency, and

$\dot{R}$ = range-rate,

and where the receiver and transmitter are collocated.

To include the plasma medium, Eq. (4a) is modified to be

$$\omega_R \approx \omega_T \left[1 - \frac{2}{c} \frac{d}{dt} \int_0^R n \; dr\right], \tag{4b}$$

where

n = index of rafraction.

For the range of integration 0 to $(R - S)$, $n = 1$ (neglecting atmospheric refraction). For the range of integration $(R - S)$ to R, $n = (1 + P)$. If P is assumed constant with respect to space,

$$\omega_R \approx \omega_T \left[1 - \frac{2}{c} \frac{d}{dt} (R + SP)\right], \tag{4c}$$

and

$$\omega_R \approx \omega_T \left[1 - \frac{2}{c} (\dot{R} + \dot{S}P + S\dot{P})\right] \tag{5}$$

Since, in practice, the range-rate is obtained from Eq. (5) with P, S = 0, then the range-rate error is

$$\Delta \dot{R} = - (\dot{S}P + S\dot{P}) \; . \tag{6}$$

Differentiation of Eq. (3) with respect to time also yields Eq. (6).

Thus, we see that the plasma contributes to the range-rate error in two ways; by changing plasma characteristics $(\dot{P})$, and by changing plasma thickness $(\dot{S})$.

CW ANGLE MEASUREMENTS

CW angle measurement systems utilize an interferometer to obtain phase difference measurements, which are then transformed into angle measurements. Two antennas, located at the ends of a baseline of known length D, each receive the same tracking signal but with a difference in phase. The path length from the vehicle to one antenna differs from the path length from the vehicle to the other antenna by a path length difference L. The phase difference $\Delta\varphi$ is proportional to L. The direction cosine, $\cos\theta$, is approximately equal to L/D. If the paths to the two antennas both traverse the same plasma then there is no angle error, since both signals have the same plasma-induced phase shift and this is cancelled by the interferometer. However, if the paths do not traverse the same plasma, there is an angle error.

The phase at antenna (1) is

$$\varphi_1 = \omega\left[t - \frac{2}{c}\left(R_1 + S_1 P_1\right)\right], \tag{7a}$$

and, at antenna (2),

$$\varphi_2 = \omega\left[t - \frac{2}{c}\left(R_2 + S_2 P_2\right)\right]. \tag{7b}$$

The phase difference at the two antennas results in a path length difference

$$L = R_2 - R_1 = \frac{1}{2}\,\frac{c}{\omega}\left(\varphi_1 - \varphi_2\right) + \left(S_1 P_1 - S_2 P_2\right) \tag{7c}$$

The direction cosine is

$$\cos\ \theta \ \approx\ \frac{L}{D} \tag{7d}$$

and

$$\cos\theta \ \approx\ \frac{1}{2}\ \frac{c}{\omega}\ \frac{\left(\varphi_1 - \varphi_2\right)}{D} + \frac{\left(S_1\ P_1 - S_2\ P_2\right)}{D}\ . \tag{7e}$$

As discussed previously, if the plasma traversed is the same for the two antennas, then the last term in Eq. (7e) is zero. However, if the plasma traversed is different, then the error $\Delta \cos\ \theta$ in the direction cosine is

$$\Delta\ \cos\ \theta \approx \frac{S_1\ P_1 - S_2\ P_2}{D}\ . \tag{8}$$

PULSE MEASUREMENTS

In pulse radar range measurements, the fundamental measurement is that of transit time. An identified pulse of radio energy travels from the radar to the vehicle, and is reflected or retransmitted to the ground station. The round-trip transit time is then transformed into range measurement. The R is related to the round-trip transit time t as

$$R = \frac{Vt}{2}\ , \tag{9a}$$

where

V = velocity of the radar pulse.

There are three distinct types of velocities associated with an electromagnetic wave. These are phase, group, and signal velocities. In free space, all three velocities are identical and equal to the velocity of light c.

Of course, the signal velocity is always less than or equal to c. For the plasma cases which we shall consider, the group and signal velocities are equivalent, and it is this velocity that must be used in Eq. (9a).

The group velocity V_g is defined as

$$V_g = \frac{d\omega}{d\beta} , \tag{9b}$$

where

$$\beta = \frac{\omega}{c} (1 + P) = \frac{\omega}{c} n . \tag{9c}$$

efore,

$$V_g = \frac{c}{n + \omega \; dn/d\omega} . \tag{9d}$$

Since the round-trip transit time is given by

$$t = 2 \left[\frac{R - S}{c} + \frac{S}{V_g} \right], \tag{9e}$$

then the range is

$$R = \frac{ct}{2} - S\left(n-1 + \omega \frac{dn}{d\omega} \right) . \tag{9f}$$

Since $n = (1 + P)$,

$$R = \frac{ct}{2} - S\left(P + \omega \frac{dP}{d\omega} \right). \tag{9g}$$

In practice, Eq. (9g) is used to obtain the range with $S = 0$. Therefore, the range error is

$$\Delta R = - S\left(P + \omega \frac{dP}{d\omega}\right). \tag{10}$$

The error in the transit-time measurement of range differs from that of the CW measurement by the dispersive characteristics of the plasma, the $\omega \; dP/d\omega$ term. Compare Eqs. (10) and (3).

SECTION III

APPLICATIONS

GENERAL

The two major sources of plasma-induced electronic tracking errors which we shall consider are reentry plasma and flame plasma. For the purpose of a first-order investigation of error effects, reentry and flame plasma are considered to differ only in the thickness of the plasma and in the time dependency of the plasma properties. The difference is, of course, much more complex. However, the approach used is sufficient to illustrate the first-order magnitude of errors which are possible.

During certain portions of the trajectory, blackout will occur and tracking will be lost. No signal is received, and, therefore, no errors are produced. Since we are concerned with the errors in tracking, a substantial signal must be received. Therefore, we will focus our attention on those portions of the trajectory which allow a successful tracking operation. Since we are concerned with present tracking accuracies, we will not consider techniques for reducing detrimental plasma effects (see Reference 1). For a successful tracking operation, the "Permissible Condition" must hold. From Reference 1, the Permissible Condition is

$$\omega_p^2 \, \omega_c \leq \omega \left(\omega^2 + \omega_c^2\right), \tag{11}$$

$$\omega_p^2 < \omega^2 + \omega_c^2 ,$$

where

ω_p = angular plasma frequency,

ω_c = collision frequency, and

ω = angular signal frequency.

In Section II, the error equations were derived. The errors were found to be functions of signal frequency ω, baseline length D, plasma thickness S, phase shift factor P, and appropriate derivatives of these parameters. The signal frequency and baseline length are fixed by the tracking system. The phase shift factor, from Reference 1, is

$$P \approx \frac{1}{2} A^2 - A\left(\frac{\omega}{\omega_c}\right) < 1 \ , \tag{12}$$

where

$$A \approx \frac{1}{2}\left(\frac{\omega_p^2}{\omega^2 + \omega_c^2}\right)\left(\frac{\omega_c}{\omega}\right) < 1 \ . \tag{13a}$$

The dispersion term is

$$\omega \frac{dP}{d\omega} \approx \omega \frac{dA}{d\omega}\left[A - \left(\frac{\omega}{\omega_c}\right)\right] - A\left(\frac{\omega}{\omega_c}\right) \ , \tag{13b}$$

where

$$\omega \frac{dA}{d\omega} \approx - A\left[1 + \frac{2\omega^2}{\omega^2 + \omega_c^2}\right] \ , \tag{13c}$$

and, therefore,

$$\omega \frac{dP}{d\omega} \approx 2A \left(\frac{\omega}{\omega_c}\right)\left(\frac{\omega}{\omega^2 + \omega_c^2}\right) - A^2 \left(1 + \frac{2\omega^2}{\omega^2 + \omega_c^2}\right). \qquad (14)$$

If an electronic tracking system operates successfully in spite of the presence of plasma, Eqs. (12) and (14) are the phase and dispersion contributions to the first-order errors generated.

We may now proceed to the determination of the possible first-order error magnitudes produced by the presence of reentry and flame plasma.

REENTRY PLASMA

We shall first investigate the plasma-generated electronic tracking errors during reentry. Those portions of the trajectory which are blacked out usually have plasma characteristics such that $| P, A | > 1$ (see Reference 1). Successful tracking usually implies $| P, A | < 1$. Therefore, we will approach the accuracy analysis from the viewpoint that successful tracking and the production of errors start at $| P, A | < 1$, with maximum errors occurring at $| P, A \approx 1$. Using $| P, A | \lessapprox 1$ frees us, in general, from the necessity of considering the plasma properties as a function of the trajectory. Thus, the error magnitudes obtained will not be related to any specific trajectory point. Rather, these errors are indicative of those that may occur before and after the blackout period as well as during other trajectory portions where plasma exists and tracking is successful.

For successful tracking, we usually have $\omega > \omega_p, \omega_c$ (see Reference 1). A typical value for the plasma thickness is $S_0 \approx 1.0$ feet.

We may now proceed with the determination of the errors.

CW Range Measurements

The plasma distance S traversed by the signal is generally greater than the plasma thickness S_0. For the geometry illustrated in Fig. 1,

$$S = \frac{S_0}{\cos \theta} . \tag{15a}$$

Substitution of Eq. (15a) into (3) yields the range error

$$\Delta R = - \frac{S_0}{\cos \theta} P . \tag{15b}$$

For $S_0 \approx 1.0$ feet and $\theta = 60$ degrees, the range error is

$$\Delta R \approx - 2 P \lessapprox | 2 \text{ ft} | . \tag{15c}$$

Doppler Range - Rate Measurements

The plasma thickness will change as the plasma builds up around the vehicle. However, as viewed by the ground station, it will also change as the aspect angle changes. We shall investigate this latter change, which is depicted in Fig. 1.

From Fig. 1 we have

$$R \sin \theta = h, \tag{16a}$$

$$S \cos \theta = S_0 . \tag{16b}$$

For the vehicle descending straight down,

$$\dot{R} = \dot{h} \sin \theta \tag{16c}$$

and

$$\frac{\dot{S}}{S} = \frac{\dot{R}}{R} . \tag{16d}$$

Combining (16c) and (16d), we have

$$\dot{S} = \frac{S}{R} \dot{h} \sin \theta . \tag{16e}$$

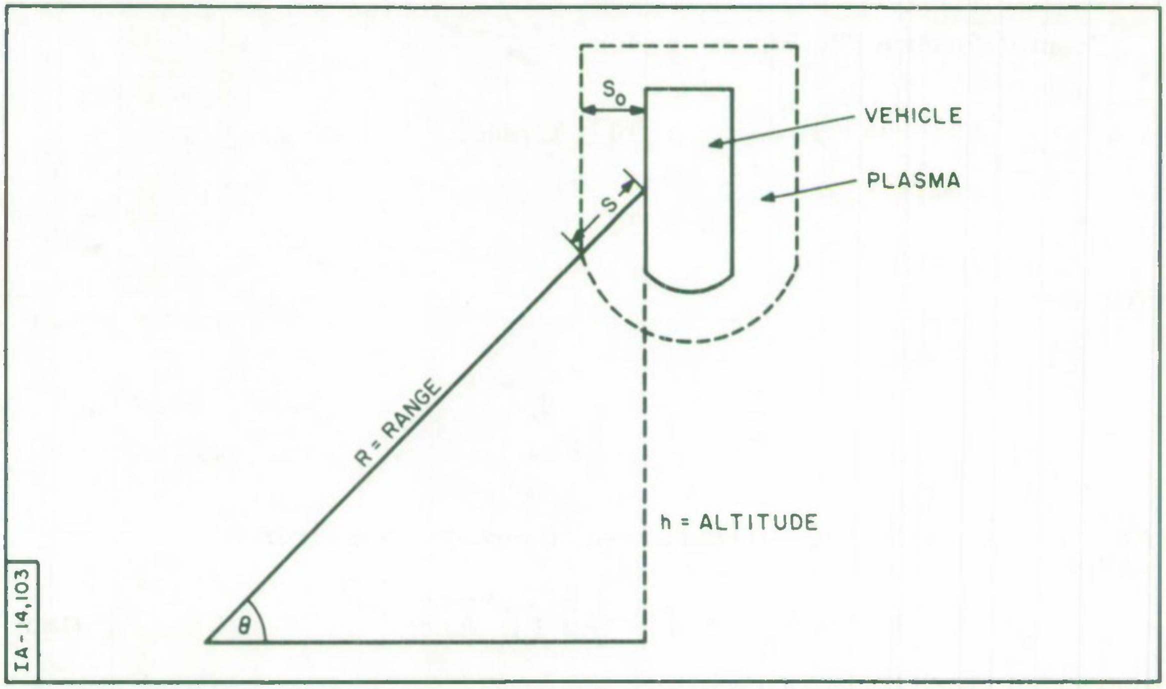

Fig. 1. Reentry Geometry for CW Range, Doppler Range-Rate and Pulse

The time derivative of the phase factor is

$$\dot{P} = \frac{dP}{dh} \dot{h} . \tag{17a}$$

From Eq. (12), with $\omega > \omega_p, \omega_c$, we find that

$$\dot{P} \approx 2 \frac{P}{\omega_p} \frac{d\omega_p}{dh} \dot{h} . \tag{17b}$$

Substitution of Eqs. (17b), (16e), and (16a) into Eq. (6) yields

$$\Delta \dot{R} \approx - \frac{S_0}{h \cos \theta} P \dot{h} \left[\sin^2 \theta + 2 \frac{h}{\omega_p} \frac{d\omega_p}{dh} \right] . \tag{18a}$$

From Reference (3), Figure 18, for

$$\dot{h} = 1.5 \times 10^4 \text{ ft./sec.},$$

$$h = 2.5 \times 10^5 \text{ ft.},$$

we find that

$$\frac{h}{\omega_p} \frac{d\omega_p}{dh} \approx -4.5 .$$

For $S_0 \approx 1.0$ feet and $\theta \approx 60$ degrees, the range-rate error is

$$\Delta \dot{R} \approx 1.0 \, P \lessapprox \left| 1 \text{ ft./sec.} \right| . \tag{18b}$$

CW Angle Measurements

Since we are concerned with a homogeneous plasma, the path length plasma variation which we will consider is as shown in Fig. 2. From the figure

$$\frac{S_2}{S_1} = \frac{R_2}{R_1} , \tag{19a}$$

and

$$S_1 - S_2 = S_1 \left(\frac{R_1 - R_2}{R_1} \right) = - S_1 \frac{L}{R_1} . \tag{19b}$$

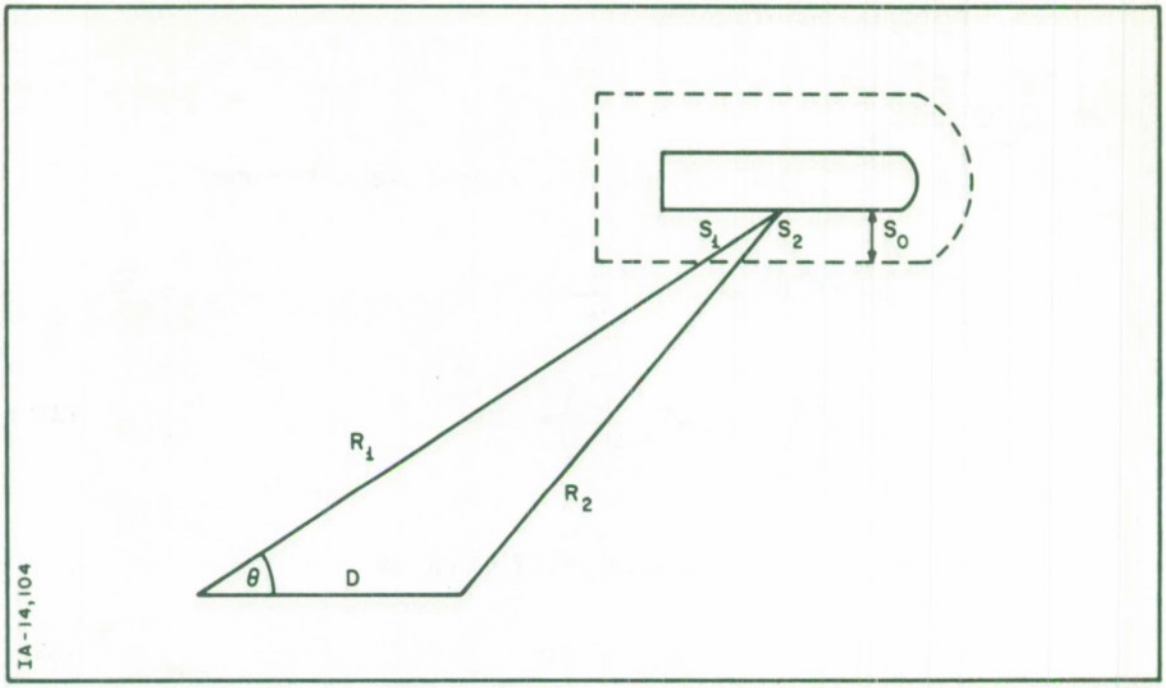

Fig. 2. Reentry Geometry for CW Angle

Substitution of Eqs. (19b) and (7d) into Eq. (8) yields

$$\frac{\Delta \cos\theta}{\cos\theta} \approx - P \frac{S_1}{R_1} , \tag{20a}$$

where

$$P_1 = P_2 = P.$$

Equation (20a) is the fractional error in the direction cosine.

For $R_1 \approx 10^6$ feet and $S_1 \approx 2$ feet,

$$\frac{\Delta \cos\theta}{\cos\theta} \approx - P \; 2 \times 10^{-6} \lessapprox \left| \, 2 \times 10^{-6} \text{ ft} \, \right| . \tag{20b}$$

Pulse Measurements

For $\omega > \omega_p, \ \omega_c$, Eqs. (12) and (14), respectively, become

$$P \approx - A \left(\frac{\omega}{\omega_c}\right),$$

$$\omega \frac{dP}{d\omega} \approx 2A \left(\frac{\omega}{\omega_c}\right) \approx -2P \ . \tag{21b}$$

Substitution of Eqs. (21a) and (21b) into (10) yields

$$\Delta R \approx SP. \tag{22}$$

The pulse and CW range errors differ in sign and are approximately of equal magnitude. Compare Eqs. (22) and (3).

For the geometry depicted in Fig. 1, we again have

$$\Delta R \approx \frac{S_0}{\cos \theta} = P \ . \tag{23a}$$

For $S_0 \approx 1.0$ feet and $\theta = 60$ degrees, the range error is

$$\Delta R \approx 2P \lessapprox |2 \text{ ft.}| \ . \tag{23b}$$

FLAME PLASMA

The ionization in the exhaust consists of thermal ionization, chemi-ionization, impact ionization, etc. Because of the multitude of ionization mechanisms, there is considerable difficulty in determining the plasma plume properties as a function of trajectory. At present most of the available information is concerned with thermal ionization, which is a principal contributor to the

total ionization. We shall, therefore, limit our investigation to thermal ionization. The errors obtained will, therefore, be on the low side, since the other ionization mechanisms will also contribute to the tracking error.

In addition, as in the reentry plasma section, we shall be concerned with successful tracking. Therefore, the third paragraph on Page 13 applies here also, and reference should be made to it.

A characteristic plasma radial thickness is $S_0 \approx 25$ feet.

We now proceed with the error determination.

CW Range Measurements

The path length thickness through the plume is assumed, for simplicity, as depicted in Fig. 3. From Fig. 3 we have

$$S = \frac{S_0}{\cos\theta} . \tag{24a}$$

Substitution of Eq. (24a) into (3) yields the range error

$$\Delta R = - \frac{S_0}{\cos\theta} P . \tag{24b}$$

For $S_0 \approx 25$ feet and $\theta = 60$ degrees, the range error is

$$\Delta R \approx - S_0 P \lesssim | 50 \text{ ft.} | . \tag{24c}$$

Doppler Range-Rate Measurements

The path length through the plume will change as the aspect angle $(\dot{S}_\theta)$ changes and as the plume volume $(\dot{S}_h)$ changes with altitude due to decreasing ambient pressure. The $\dot{S}$ term, therefore, is

$$\dot{S} = \dot{S}_\theta + \dot{S}_h . \tag{25a}$$

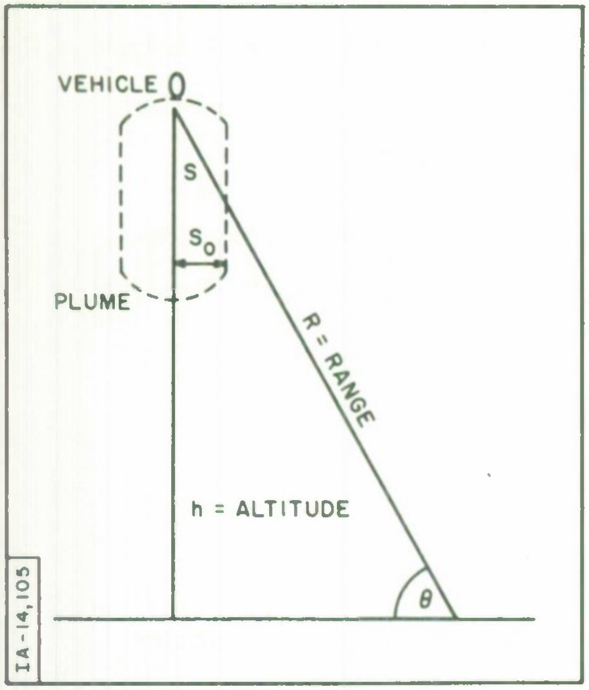

Fig. 3. Flame Geometry for CW Range, Doppler Range-Rate and Pulse

For the simple geometry depicted above, Eq. (16e) applies, and we have

$$\dot{S}_\theta = \frac{\dot{S}}{R} h \sin \theta . \tag{25b}$$

To obtain $\dot{S}_h$, we let the plume volume F be any volume, such that

$$F = a S_0^m , \tag{25c}$$

where a and m are constants. The time derivative is

$$\dot{F} = m \frac{F}{S_0} \dot{S}_0 . \tag{25d}$$

For thermal ionization, the total electron content of the plume remains approximately constant as the plume expands. The electron density, of course, changes as the plume expands. Since the electron density is proportional to ω_p^2 (see Reference 1), we have

$$\omega_p^2 F = \text{constant}. \tag{25e}$$

Use of the time derivatives yields

$$2 \dot{\omega}_p F = - \omega_p \dot{F} . \tag{25f}$$

The combination of Eq. (25f) with (25d) yields

$$\dot{S}_0 = - 2 \frac{S_0}{m} \frac{\dot{\omega}_p}{\omega_p} . \tag{25g}$$

From Eq. (12), with $\omega > \omega_p , \omega_c$, we find that

$$\dot{P} \approx 2P \frac{\dot{\omega}_p}{\omega_p} . \tag{25h}$$

Substitution of Eq. (25h) into (25g) yields

$$\dot{S}_0 = - \frac{S_0}{m} \frac{\dot{P}}{P} . \tag{25i}$$

From Fig. 3, we have

$$S \cos \theta = S_0 , \tag{25j}$$

and, holding θ = constant,

$$\dot{S}_h \cos \theta = \dot{S}_0 , \tag{25k}$$

and, from Eq. (25i),

$$\dot{S}_h \cos \theta = - \frac{S_0}{m} \frac{\dot{P}}{P} . \tag{25l}$$

Finally, we substitute Eqs. (25l) and (25b) into (25a) to obtain

$$\dot{S} \approx \frac{S}{R} \dot{h} \sin \theta - \frac{S_0}{m \cos \theta} \frac{\dot{P}}{P} . \tag{26}$$

The time derivative of the path length through the plume consists of the aspect angle change (the $\dot{h}$ portion) and the plasma expansion change (the $\dot{P}$ portion).

The next step is to determine $\dot{P}$. Since, from Eqs. (25h) and (25f),

$$\dot{P} \approx -P \frac{\dot{F}}{F} , \tag{27a}$$

we may write

$$\dot{P} \approx -P \frac{1}{F} \frac{dF}{dp} \frac{dp}{dh} \dot{h}, \tag{27b}$$

where p = ambient pressure.

Utilizing the well-known relationship

$$\frac{dp}{dh} = -g\rho , \tag{27c}$$

where

g = gravity acceleration, and

ρ = ambient density,

we have

$$\dot{P} \approx P \frac{1}{F} \frac{dF}{dp} g \rho \dot{h} . \tag{28}$$

Substitution of Eqs. (28) and (26) into Eq. (6) yields the range-rate error (using $R \sin\theta = h$, $S \cos\theta = S_0$) :

$$\Delta \dot{R} \approx - \frac{S_0 P \dot{h}}{\cos\theta} \left\{ \frac{\sin^2\theta}{h} + \left(\frac{m-1}{m} \right) \frac{1}{F} \frac{dF}{dp} g \rho \right\} . \tag{29a}$$

Let us consider volumes for which $m = 3$. For $\theta = 60$ degrees and $S_0 \approx 25$ feet, Eq. (29a) becomes

$$\Delta \dot{R} \approx - 50\, P \dot{h} \left\{ \frac{3}{4h} + \frac{2}{3} \frac{1}{F} \frac{dF}{dp} g\rho \right\} . \qquad (29b)$$

From Reference (4), Fig. 6, for h $\approx 10^5$ feet,

$$\frac{1}{F} \frac{dF}{dp} \approx - 0.7 \times 10^{-2} \text{ per lb./ft.}^2 ,$$

and since

$$g \approx 32 \text{ ft./sec.}^2, \text{ and}$$

$$\rho \approx 3.3 \times 10^{-5} \text{ slugs/ft.}^3 ,$$

the range-rate error for $\dot{h} \approx 1.5 \times 10^4$ ft./sec. is

$$\Delta \dot{R} \approx - 1.9\, P \lesssim | 1.9 \text{ ft./sec.} | . \qquad (29c)$$

CW Angle Measurements

Since we are concerned with a homogeneous plasma, the plume path length variation which we will consider is depicted in Fig. 4. To a first approximation, Eq. (20a) applies:

$$\frac{\Delta \cos \theta}{\cos \theta} \approx - P \frac{S_1}{R_1} . \qquad (30a)$$

Equation (30a) is the fractional error in the direction cosine.

The assumption that $S_1 \approx 50$ feet yields, for $R_1 \approx 10^6$ feet,

$$\frac{\Delta \cos \theta}{\cos\theta} \approx - P \times 5 \times 10^{-5} \lesssim | 5 \times 10^{-5} | . \qquad (30b)$$

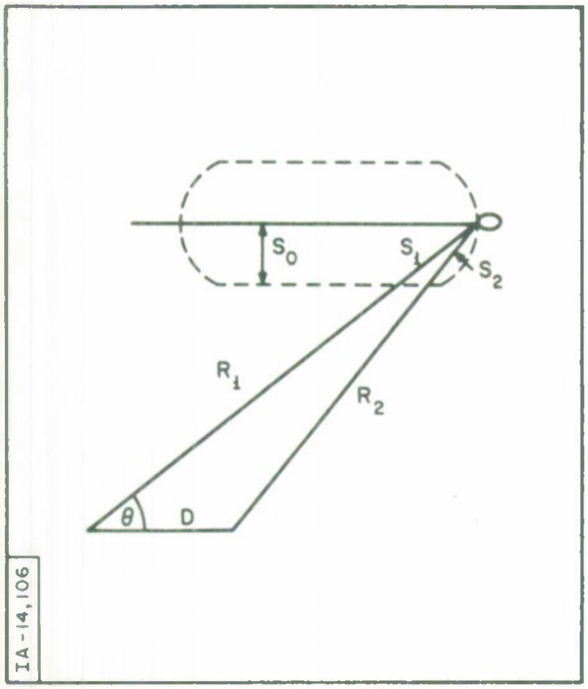

Fig. 4. Flame Geometry for CW Angle

Pulse Measurements

Equations (21) and (22) and the discussion on Page 13 apply directly. From Eq. (22) we have

$$\Delta R \approx SP . \tag{31a}$$

From Fig. 3 we again have

$$\Delta R \approx \frac{S_0}{\cos \theta} P . \tag{31b}$$

For $S_0 \approx 25$ feet and $\theta = 60$ degrees, the range error is

$$\Delta R \approx 50 P \lesssim |50 \text{ ft.}| . \tag{31c}$$

SECTION IV

SUMMARY AND CONCLUSIONS

We have investigated plasma-generated electronic tracking errors for both reentry and flame plasma. Simple models have been utilized to obtain order-of-magnitude errors for a number of measurement techniques. Table 1 lists the measurement systems, techniques employed, and parameters measured.

Table 1

Measurement Systems Investigated

System	Technique	Parameter
CW range	Round-trip phase displacement	Range
Doppler	Frequency shift	Range-rate
CW angle	Path length baseline phase comparison	Direction cosine
Pulse	Round-trip transit time	Range

The CW range error results from the plasma-induced phase shift. The Doppler range-rate error results from the time rate of change of the plasma properties and the time rate of change of the aspect angle. The CW angle error results from the different plasma path lengths to the ends of the baseline. The pulse range error results from the increased transit time produced by the presence of the plasma.

Table 2 summarizes the possible error magnitudes for the simple models investigated.

Table 2

Plasma Electronic Tracking Errors

System	Reentry	Flame
CW range	2.0 ft	50 ft.
Doppler	1.0 ft./sec.	1.9 ft./sec.
CW angle	$2 \times 10^{-4}\%$	$5 \times 10^{-3}\%$
Pulse	2.0 ft.	50 ft.

The CW angle error is the percentage error in the direction cosine.

For the simple model considered, the errors for flame plasma are greater than for reentry plasma; basically, for the reason that the plume is considerably larger than the plasma sheath.

With the present emphasis upon greater electronic tracking accuracies, it would appear that consideration should be given to plasma-generated errors.

Jerome Hoffman

Jerome Hoffman

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13. ABSTRACT

Simple models are utilized to calculate the order of magnitude of errors generated by reentry and flame plasma effects in typical tracking systems.

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